A MODIFIED ERGUN EQUATION FOR YIELD STRESS FLUID FLOW THROUGH PACKED BEDS

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ABSTRACT

Non-Newtonian fluid flows through packed beds occur frequently in industry. To the Authors' knowledge, correlations have not been developed for Yield Stress fluid flow through packed beds. Interest in non-Newtonian fluid flow through porous media has motivated this work to develop such a correlation for Yield Stress fluids.

The correlation is developed by introducing the Yield Stress model in place of the Newtonian model used in Ergun's Equation. The resulting model has three parameters that are functions of the geometry and roughness of the particle surfaces. Two of the parameters can be deduced in the limit as the Yield Stress become negligible and the model reduces to Ergun's Equation for Newtonian fluids. The third model parameter is determined from experimental data. The correlation relates a defined friction factor to the dimensionless Reynolds and Hedstrom numbers that can be used to predict pressure drop for flow of a yield stress fluid through a packed bed of spherical particles.

INTRODUCTION

Packed beds are widely used throughout industries for a variety of purposes. Predicting pressure drop for a given flow rate is important for the design of such processes. A number of correlations are available in literature for this purpose. Ergun [1952] combined the Blake-Kozeny expression, derived as a model for Newtonian flows through a bundle of capillary tubes, with an empirical Burk-Plummer relation, to obtain a packed bed model applicable to a wide range of Reynolds numbers.

Christopher and Middleman [1965] introduce a modified Darcy model to represent non-Newtonian flows through porous media in which a modified permeability accounts for the non-Newtonian behavior. Park *et al.* [1975] evaluated several approaches to modeling non-Newtonian flows through packed beds and concluded that the capillary tube approach was the best choice when combined with the a particular rheological expression. Marshall and Metzner [1967] discuss the effects of viscoelastic properties on flows through packed beds and the importance of the Deborah number for such fluids. Hayes *et.al.* [1996] model the flow of power law fluids through packed beds from a volume averaged approach and account for wall effects. Shirato *et.al.* [1980] demonstrate the effects of non-Newtonian behavior on cake filtration.

To our knowledge, no one has developed a packed bed model specifically for a yield stress fluid. In this paper the bundle of capillary tubes approach is applied to model the yield stress fluid through a porous medium. This model is combined with the Burk-Plummer relation to obtain the analogous Yield Stress modified Ergun Equation.

ERGUN'S EQUATION FOR NEWTONIAN FLUIDS

The Ergun's equation for Newtonian fluid flow through a packed bed that is most often reported in literature has the form (Bird *et.al.*, 1960)

$$\frac{P_0 - P_L}{L} = \frac{150\mu V}{d_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\rho V^2}{d_p}\frac{(1-\varepsilon)}{\varepsilon^3} \tag{1}$$

To help explain the derivation of the modified Ergun Equation for Yield Stress fluids, a derivation of the Ergun Equation for Newtonian Fluids is first summarized. The form of the derived Ergun equation form is modified to allow direct comparison with the model developed for the Yield Stress fluid. The friction factor, f, is defined by the rate expression (Bird *et.al.*, 1960) for flow in a circular tube

$$F_k = f(2\pi RL)\left(\tfrac{1}{2}\rho\langle v\rangle^2\right) \tag{2}$$

where F_k is the drag force along the tube wall and $\langle v\rangle$ is the average velocity of the fluid through the tube. A force balance on the fluid over the length of the tube relates the drag force to the pressure drop along the length of the tube

$$F_k = \pi R^2(P_0 - P_L) \tag{3}$$

Combining Eqs.(2) and (3) to eliminate the drag force gives a working expression for the friction factor

$$f = \frac{R}{L}\frac{(P_0 - P_L)}{\rho\langle v\rangle^2} \tag{4}$$

For flow in the packed bed, we consider the bed to be a bundle of capillary tubes of equal diameters and equal flow rates through each tube. The total flow rate through all N tubes is

$$Q = N(\pi R_{tube}^2)\langle v\rangle \tag{5}$$

Also, the total flow rate is related to the bed average velocity, V, by

$$Q = \pi R_{bed}^2 V \tag{6}$$

Eliminating the flow rates between Eqs.(5) and (6) gives

$$\begin{aligned}\langle v\rangle &= \frac{\pi R_{bed}^2}{N\pi R_{tube}^2}V \\ &= \frac{\pi R_{bed}^2 L}{N\pi R_{tube}^2 L}V \\ &= \frac{1}{\varepsilon}V\end{aligned} \tag{7}$$

where the porosity, ε, equals the ratio of the total volume of the capillary tubes divided by the volume of the bed.

Knowing that the bed actually consists of spherical particles of diameter d_p, and not capillary tubes, we introduce the hydraulic radius. The radius of the tubes modeled in Eq.(5) is related to the hydraulic radius by

$$R_{tube} = 2R_h \tag{8}$$

However, the hydraulic radius is also related to the porosity and the particle diameter by

$$
\begin{aligned}
R_h &= \left(\frac{\text{Area available for flow}}{\text{WettedPerimeter}}\right) \\
&= \left(\frac{\text{Volume available for flow}}{\text{Wetted surface area}}\right) \\
&= \frac{\left(\frac{\text{Volume of voids}}{\text{Volume of Bed}}\right)}{\left(\frac{\text{Wetted Surface}}{\text{Volume of Bed}}\right)} \\
&= \frac{\varepsilon}{a}
\end{aligned}
\tag{9}
$$

The bed-specific surface area, a, is related to the specific surface area per volume of particles by

$$
a = a^s(1-\varepsilon) \tag{10}
$$

and the latter is related to the particle diameter, d_p, by

$$
d_p = \frac{6}{a^s} \tag{11}
$$

Combining Eqs.(9)-(11) we get the hydraulic radius as

$$
R_h = \frac{\varepsilon d_p}{6(1-\varepsilon)} \tag{12}
$$

Finally, combining Eqs.(4), (7), (8), and (12) we get the expression for the friction factor in terms of the particle diameter as

$$
f = \frac{\varepsilon^3 d_p}{3(1-\varepsilon)L}\frac{(P_0-P_L)}{\rho V^2} \tag{13}
$$

Equation (13) applies for all flow regimes, large or small Reynolds numbers. The Reynolds number for flow in the capillary,

$$
R_e = \frac{\rho\langle v\rangle 2R_{Tube}}{\mu} \tag{14}
$$

is related to the packed bed by combining Eqs.(14) with (7), (8), and (12) to obtain

$$
R_e = \frac{\rho V d_p}{\mu}\frac{4}{6(1-\varepsilon)} \tag{15}
$$

We define the Reynolds number for the packed bed as

$$
R_{ep} = \frac{\rho V d_p}{\mu(1-\varepsilon)} \tag{16}
$$

hence

$$
R_e = \tfrac{2}{3}R_{ep} \tag{17}
$$

For laminar flow we introduce the laminar flow friction factor correlation for the flow in a capillary.

$$f_0 = \frac{16}{R_e} \tag{18}$$

where f_0 is the friction factor value at low Reynolds number. Combining Eqs.(17) and (18) we get

$$f_0 = \frac{24}{R_{ep}} \tag{19}$$

When we eliminate the friction factor and Reynolds numbers between Eqs.(19), (16), and (13) we obtain

$$\frac{(P_0 - P_L)}{L} = \frac{72\mu V}{d_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} \tag{20}$$

In laminar flow, the assumption of hydraulic radius frequently gives too large a flow rate for a given pressure gradient. Hence, the number 72 is expected to be too small. Analysis of experimental data led to improvement of the formula by replacing the 72 in the denominator of Eq.(20) with 150, and yields the Blake-Kozeny equation

$$\frac{(P_0 - P_L)}{L} = \frac{150\mu V}{d_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} \tag{21}$$

For large Reynolds numbers, a similar analysis with experimental data produced what is known as the Burke-Plummer equation, in the form

$$\frac{(P_0 - P_L)}{L} = 1.75\frac{1}{d_p}\rho V^2 \frac{1-\varepsilon}{\varepsilon^3} \tag{22}$$

Combining Eqs.(22) and (13) yields the friction factor for large Reynolds numbers to be

$$f_\infty = 0.5833 \tag{23}$$

Ergun [1952] found that by adding Eqs.(21) and (22) we obtain a correlation for the full range of flows, as given in Eq.(1). This is equivalent to summing the two asymptotic solutions to obtain the friction factor for the full range of Reynolds Numbers as

$$f = f_0 + f_\infty \tag{24}$$

MacDonald *et.al.* (1979) extended Ergun's results to a wider range of materials and found that the correlation is improved by replacing the 150 in Eq.(1) with 180, and by replacing the 1.75 with 1.8 for smooth particles or by 4.0 for rough particles. In this work we only consider the smooth particles, hence Eq.(1) is revised and the Ergun Equation for Newtonian fluids becomes

$$\frac{P_0 - P_L}{L} = \frac{180\mu V}{d_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.80\rho V^2}{d_p}\frac{(1-\varepsilon)}{\varepsilon^3} \tag{25}$$

For comparison with non-Newtonian fluids it is more convenient to express the correlation in the form of Eq. (24). Taking into account the refinement by MacDonald, *et.al.* (1979), the functional forms for the Low and High Reynolds Number friction factors are

$$f_0 = \frac{C_1}{\text{Re}} \qquad \text{where } C_1 = 60 \tag{26}$$

$$f_\infty = C_2 \qquad \text{where } C_2 = 0.6 \tag{27}$$

Hence, one can model the pressure drop for a Newtonian fluid flowing through a packed bed using Ergun's equation in the form of Eq.(25), or one can calculate the friction factor through Eqs.(24), (26), and (27), and then calculate the pressure drop using the definition of the friction factor, in Eq.(13). For Newtonian fluids, Eq.(25) is more direct. For the yield stress fluid it turns out that the second method, calculating the friction factor first, is the most convenient approach.

MODIFIED ERGUN'S EQUATION FOR YIELD STRESS FLUIDS

Following a similar approach for Yield Stress fluid flow through a packed bed requires (1) a laminar flow correlation for yield stress fluids in a tube, (2) asymptotic assumptions that as the yield stress goes to zero (or when the Reynolds number becomes very large), the solution collapses to the Newtonian fluid correlation, and (3) an expression for when the fluid will not flow.

The Yield Stress constitutive equation has the form

$$\tau_{rz} = -\mu_o \frac{dv_z}{dr} \pm \tau_o \qquad \text{for } |\tau_{rz}| > \tau_o$$

$$\frac{dv_z}{dr} = 0 \qquad \text{for } |\tau_{rz}| < \tau_o \tag{28}$$

The Low Reynolds Number laminar flow solution for a Yield Stress fluid flowing in a tube (Bird, *et.al.*, 1960) gives

$$\pi R_{tube}^4 \langle v \rangle = \frac{\pi (P_0 - P_L) R_{tube}^4}{8 \mu_o L} \left[1 - \frac{4}{3}\left(\frac{\tau_o}{\tau_R}\right) + \frac{1}{3}\left(\frac{\tau_o}{\tau_R}\right)^4 \right] \tag{29}$$

where τ_R is the shear stress value at the tube wall.

Equation (29) contains the yield stress, τ_o. The dimensionless group that represents the yield stress is the Hedstrom number. We define the packed bed Hedstrom number as

$$H_{ep} = \frac{\tau_o \rho d_p^2}{\mu_o^2} \frac{\varepsilon^2}{(1-\varepsilon)^2} \tag{30}$$

Combining Eqs.(7), (8), (12), (13), (29), and (30), with some algebraic rearrangement, we derive the low Reynolds Number friction factor value to be

$$f_0 = \frac{24}{R_{ep}} \left[1 - \frac{4}{3}\left(\frac{2H_{ep}}{f_0 R_{ep}^2}\right) + \frac{1}{3}\left(\frac{2H_{ep}}{f_0 R_{ep}^2}\right)^4 \right]^{-1} \tag{31}$$

The assumption of hydraulic radius is expected to make the numerical values of 24 and 2 in Eq.(31) to be too small, as was the case with the Newtonian fluid. Those constants are replace with constants C_1 and C_3, to be determined.

$$f_0 = \frac{C_1}{R_{ep}} \left[1 - \frac{4}{3}\left(\frac{C_3 H_{ep}}{f_0 R_{ep}^2}\right) + \frac{1}{3}\left(\frac{C_3 H_{ep}}{f_0 R_{ep}^2}\right)^4 \right]^{-1} \tag{32}$$

Since the effect of the Yield Stress diminishes as Reynolds Number becomes very large (*ie.* for large strain rates) the Yield Stress fluid is expected to behave similar to a Newtonian fluid at Large Reynolds Number. Hence, for Large Reynolds Numbers, the friction factor becomes

$$f_\infty = C_2 \tag{33}$$

Following Ergun's approach, the friction factor for the full flow range takes the form

$$f = f_0 + f_\infty \tag{34}$$

where the constants C_1, C_2, C_3 in Eqs. (32) and (33) must be determined.

In the limit as $\tau_o \to 0$, Eq.(32) must reduce to the Newtonian fluid correlation, Eq.(24). Hence we conclude $C_1 = 60$ and $C_2 = 0.6$ for smooth particles. For rough particles $C_2 = 1.33$ as determine by MacDonald, *et.al* [1979]. The constant, C_3, must be determined empirically from experiments with yield stress fluids (the topic of the next section).

The inequality condition in the yield stress model, Eq. (28) suggests that if the applied pressure is not large enough, no flow will occur. For flow in a tube this occurs when $\tau_0 / \tau_R < 1$. This ratio appears on the right side of Eq.(29) and by inspection with Eq.(31), the equivalent requirement for flow is

$$\frac{C_3 H_{ep}}{f R_{ep}^2} < 1 \tag{35}$$

In measurable quantities, flow occurs when the condition

$$\frac{(P_0 - P_L)}{L} > \frac{3C_3 \tau_o}{d_p} \frac{(1-\varepsilon)}{\varepsilon} \tag{36}$$

is satisfied.

EXPERIMENTAL APPROACH

A true yield stress fluid behavior is difficult to produce in a laboratory. However, a number of fluid mixtures give good approximations to the yield stress behavior. One such fluid, as reported by Wünsch [1990], is an aqueous solution of Carbopol 941 (BF Goodrich Corporation). In the experiments reported here, solutions with concentrations varying between 0.15 to 1.3 mass percent of Carbopol 941 by mass are used to determine the parameter C_3. All rheological measurements are for the mixtures at room temperature. The rheological parameters for the characterizing the fluid are determined from shear stress – shear rate curves, obtained by a dynamic stress rheometer.

In the packed bed experiments the aqueous solution of Carbopol 941 is pumped through a packed column of glass beads in the experimental setup shown in the diagram in Figure 1. The pressure drop is measured over a range of flow rates for each concentration. The packed column is a Plexiglas tube of 5.7 cm in inside diameter and 110 cm in length. The spherical glass beads have a narrow size distribution with a number-average diameter of 0.211 cm, as determined by microscopic measurement. The tube wall taps for pressure measurements are 87 cm apart and two tube diameters from the ends of the packed bed to minimize entrance and exit effects. The column, fluid reservoir, and flow meter are mounted vertically, and fitted with pipes and valves to allow upward flow for filling and downward flow for testing. Air pressure is used to drive the flow.

The bed porosity is calculated from Ergun's Equation, Eq.(1), with pressure drop and flow rate data of distilled water flowing through the packed column. The calculated porosity of 0.37 agrees with reported values of beds with normal packed spherical particles [Foust, et.al, 1960].

RESULTS AND DISCUSSION

The rheological data for several different Carbopol 941 concentration samples are shown in Figure 2. The data show some curvature for small shear rates, but the overall behavior of the fluid gives an approximation of a yield stress fluid. The non-linear character of the data make interpretation difficult because it is not clear whether the data for high shear rates should be extrapolated to the yield stress value, or whether just the data in the low shear rates range should be used, or if it is best to fit the yield stress model to all of the data and effectively ignore the curvature. Because of the highly tortuous flow paths for flows in packed beds, one would expect that high and low shear rates are important and that the best fit of the data would fall somewhere in between the extremes. Hence, the latter approach is applied here.

The data in Figure 2 are fitted by least-squares method to a linear curve to obtain values for μ_o and τ_o from the slope and intercept. The Carbopol 941 is not easy to mix uniformly and not easy to reproduce mixtures of the same concentrations between the rheological and packed bed experiments. Because of uncertainties in the mixture concentrations the fitted values of μ_o and τ_o are plotted in Figure 3 as functions of concentration. The plots in Figure 3 in turn are fitted by least squares fit to obtain the values of μ_o and τ_o as functions of concentration. The values from these fitted curves are used in the packed bed analysis below. The curves corresponding to these modified values of μ_o and τ_o are overlaid on the experimentally measured data in Figure 2. The overlaid curves agree well with most of the experimental data. The sample concentrations and the modified μ_o and τ_o values are listed in Table 1.

The value of C_3 is determined by minimizing the error between the experimental and calculated pressure drop data in the packed bed experiments shown in Figure 4. Trial values for C_3 are plotted in Figure 5 versus the error. For each data point the error is defined as the magnitude of the difference between experimental and calculated pressures. The average error is the sum of the errors of all of the data points over the ranges of velocities and Carbopol 941 concentrations divided by the total number of data points. As Figure 5 shows, the minimum occurs at $C_3 = 3.5 \pm 0.01$.

The results in Figure 4 give a remarkably good fit of the experimental data over a range of pressures, flowrates, and material parameters. The value of $C_3 = 3.50$ is also remarkably close to the value of 2 predicted using the bundle of capillary tubes model, as shown in Eq.(31).

The friction factor may be calculated as a function of velocity using Eqs.(32) – (34) where $C_1 = 60.0$, $C_2 = 0.60$ and $C_3 = 3.50$. The pressure drop follows from the friction factor and Eq.(13). However, Eq (32) is difficult to use to predict the friction factor and pressure drop because of its implicit dependence on f_0. To plot the results in a more convenient form, Eq.(32) is rearranged in order to eliminate of friction term in denominator.

$$f_0 = \left[\left(\frac{C_1}{\mathrm{Re}_p} + \frac{4}{3}\frac{C_3 He_p}{\mathrm{Re}_p^2} \right) f_0^3 - \frac{1}{3}\left(\frac{C_3 He_p}{\mathrm{Re}_p^2} \right)^4 \right]^{0.25} \qquad (37)$$

The Reynolds and Hedstrõm numbers are calculated via Eqs. (16) and (30). A method of successive substitution is used to calculate f_0 from Eq. (37) from an initial guess and for fixed values of R_{ep} and H_{ep}. Since Eq. (37) is fourth order in f_0, the polynomial has four roots, of which two are negative and two are positive. The negative values can be discarded because the friction factor cannot be negative. For two positive values, one is larger than the corresponding value for Newtonian fluids and one is near zero. The correct value is the value greater than the value for the Newtonian fluid [Patel, 1983].

The friction factor is plotted in Figure 6 as a function of R_{ep} and H_{ep}. This correlation may be used to calculate the friction factor and the pressure subsequently calculated via Eq. (13). With this plot the pressure drop is easily calculated without the iterative calculations required by Eq. (37). The criteria given in Eq.(36) determines when flow occurs.

CONCLUSIONS

In this work, a correlation for the friction factor for flow of a Yield Stress fluid through a packed bed is derived. This correlation has three parameters, two of which are determined from the Ergun Equation for Newtonian fluids. The third parameter is determined from experimental data from aqueous solutions of Carbopol 941. The resulting correlation can be used to estimate the pressure drop for flow of a yield stress fluid through a packed bed. A criteria for determining when flow will or will not occur is also deduced.

NOTATION

a	wetted surface per volume of bed [m^2/m^3]
a^s	specific surface per volume of solids [m^2/m^3]
C_1, C_2, C_3	Constants in correlations.
d_p	particle diameter [m]
F_k	force acting on the wall cause of the fluid flow [N]
f	friction factor [-]
H_e	Hedstrom number [-]
H_{ep}	Hedstrom number for packed bed [-]
L	bed length [m]
P_0	pressure at inlet to bed [N/m^2]
P_L	pressure at exit of bed [N/m^2]
R	capillary radius [m]
R_e	Reynolds number [-]
R_{ep}	Reynolds number for packed bed [-]
R_h	hydraulic radius [m]
$\langle v \rangle$	average velocity in capillary [m/s]
V	superficial velocity [m/s]
μ_o	Yield Stress modulus [N/m^2]
τ_o	yield stress [N/m^2]
ε	porosity [-]
ρ	density [kg/m^3]

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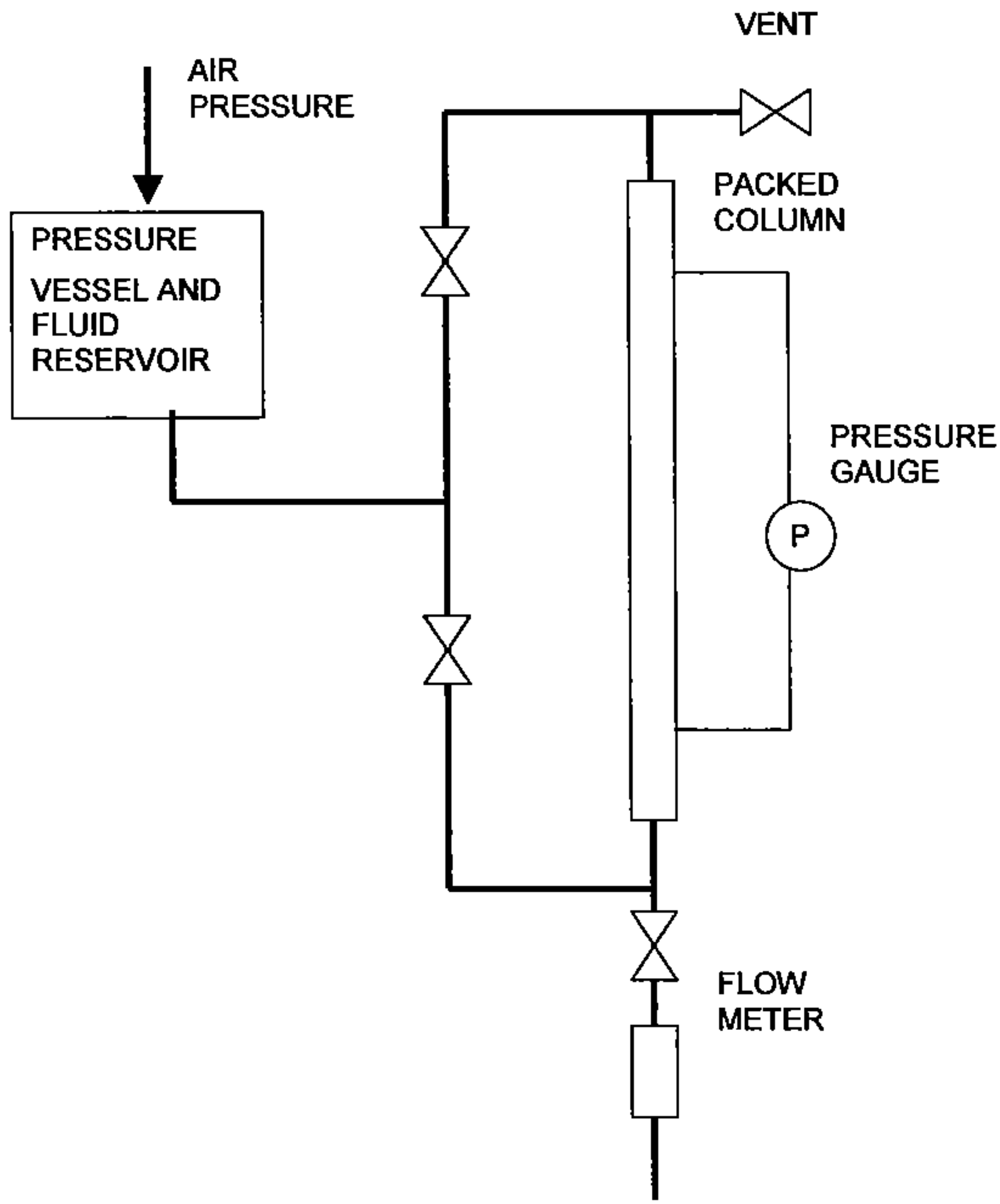


Figure 1. Schematic of the packed column experiment. Air pressure drives the fluid flow. For filling the column with fluid, the valves are positioned to drive the flow upward to vent air out of the top. In the experiments the flow is driven downward through the column and the flow meter while pressure drop and flow rate are monitored.

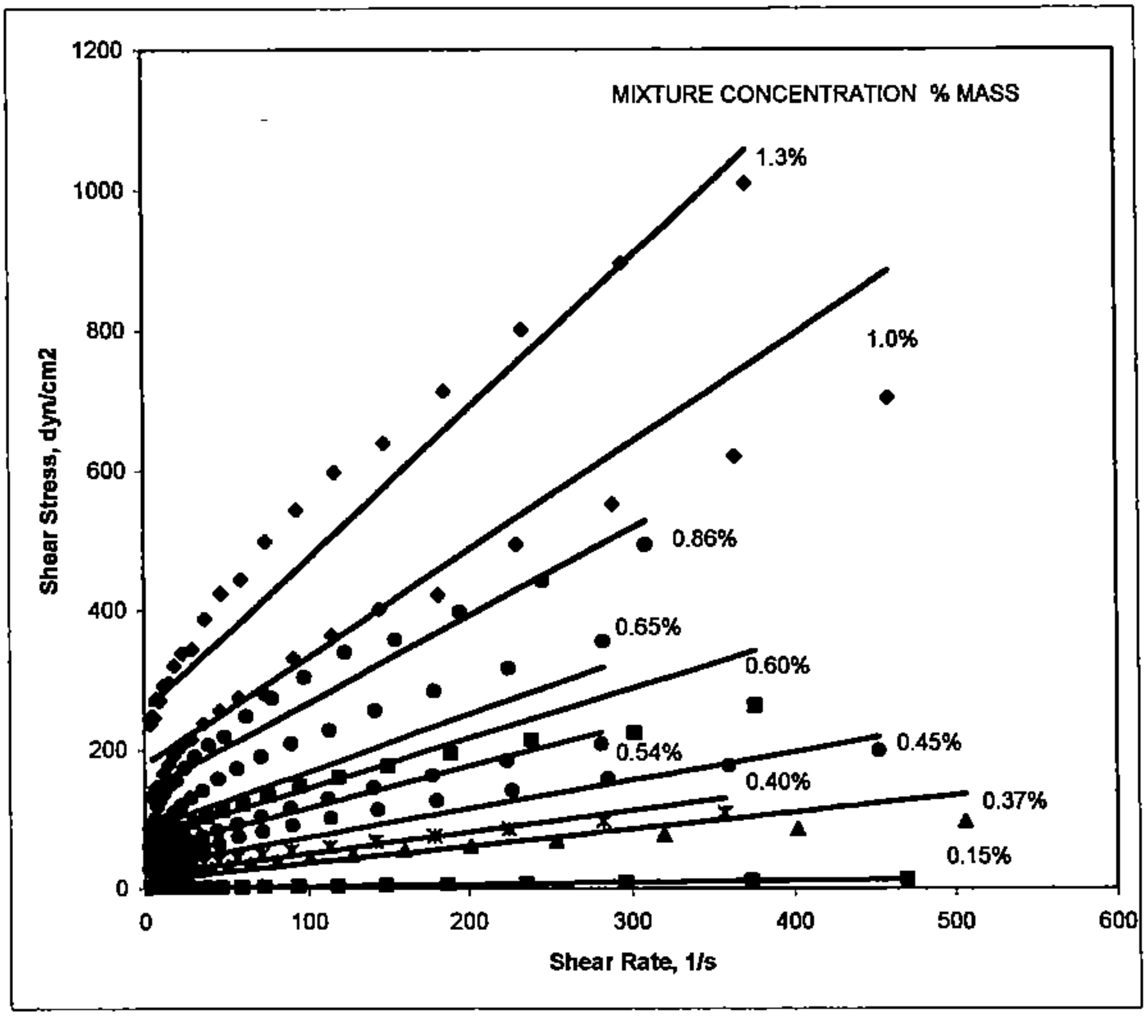
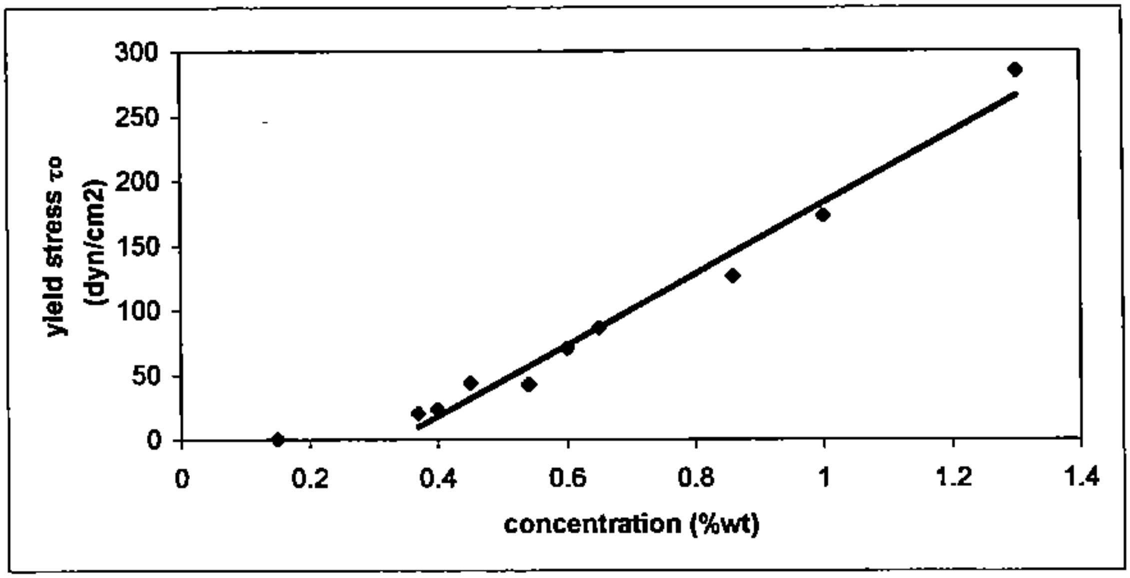
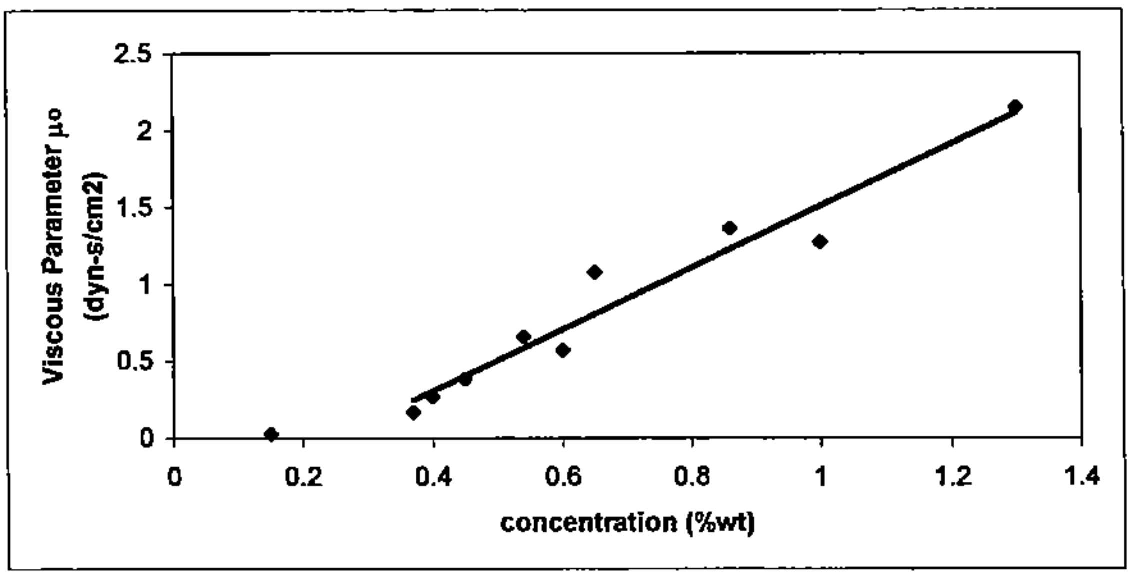


Figure 2. Shear stress versus the shear rate in rheological experiments to determine the viscous and yield stress parameters in the yield stress model. The discrete data points are the experimental values. The solid curves are the curves used to estimate the rheological parameters taken from the fitted curves in Figure 3. The concentrations are the mass fractional amounts of Carbopol 941 in aqueous solution.



(a)



(b)

Figure 3. (a) Yield stress, τ_o, versus concentration. (b) Viscosity parameter, μ_o, versus concentration. The discrete data points are the μ_o and τ_o values obtained by least squares fitting of the data in Figure 2. The solid curves are the least squares fitted values as a function of concentration and are used in the packed bed analysis. The regression coefficient for the curve in (a) is $R^2 = 0.975$ and for (b) is $R^2 = 0.954$.

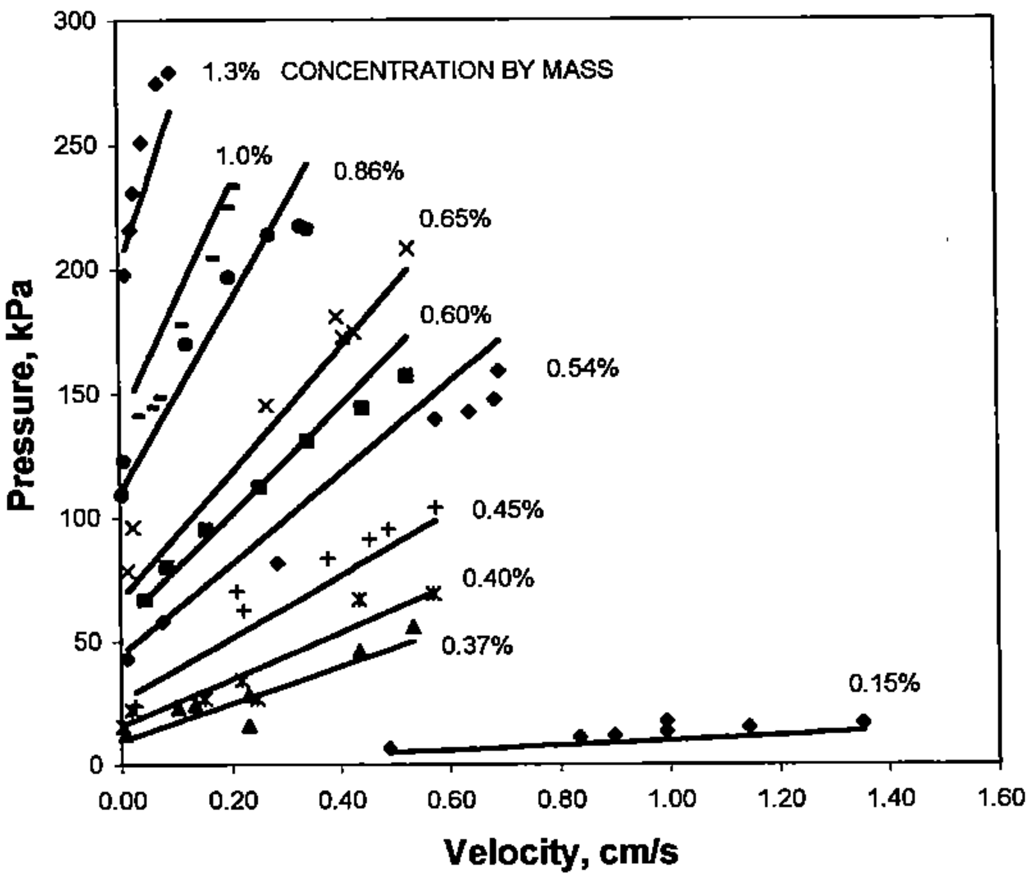


Figure 4. Pressure drop versus velocity data for flow of a yield stress fluid in a packed bed. The discrete points are experimentally measured data points. The curves are calculated values for $C_3 = 3.50$ and the yield stress model parameters, τ_0 and μ_0, determined from the fitted values listed in Table 1. The concentrations are the mass fractional amounts of Carbopol 941 in aqueous solution.

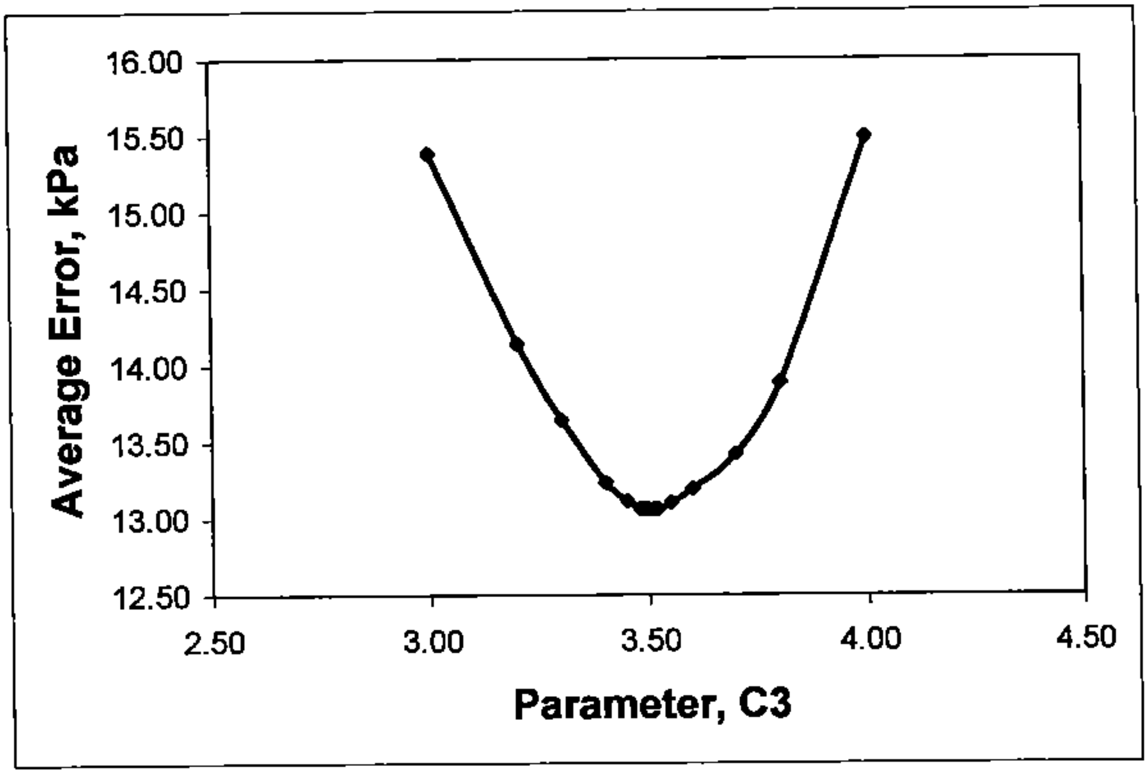


Figure 5. Trial values for parameter C_3 are plotted versus the average error between the experimental and calculated pressure drops in the packed bed experiments. The minimum error occurs with $C_3 = 3.50$.

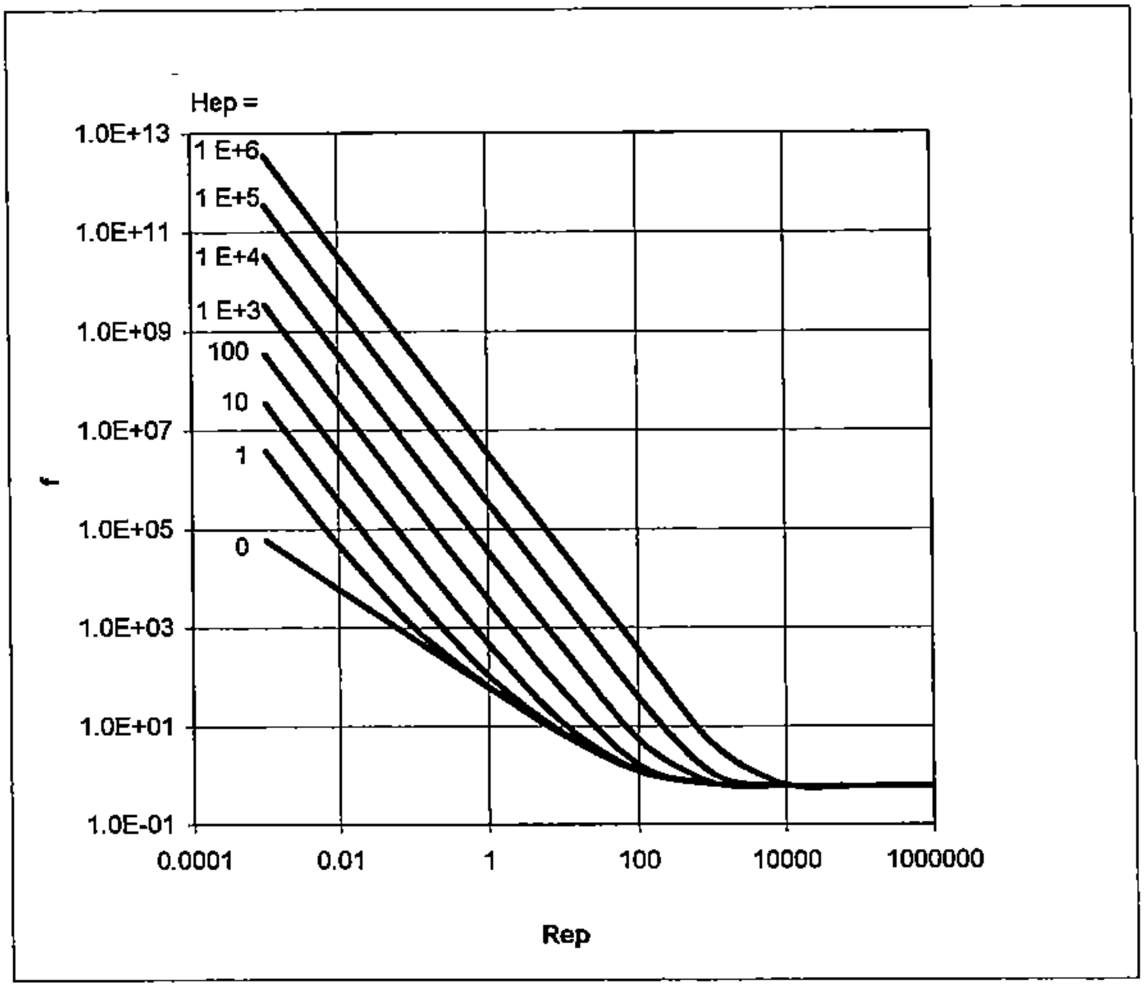


Figure 6. Friction factor plot for yield stress fluid flow through a packed bed. The friction factor is plotted from Eqs. (32)-(34) and for values $C_1 = 60.0$, $C_2 = 0.60$, and $C_3 = 3.50$. The curve for $H_e = 0$ is equivalent to Ergun's Equation for a Newtonian Fluid.

Table 1. Yield stress and Viscous parameters for fluid samples as functions of solution concentrations of Carbopol 941.

Sample no.	Concentration, (mass percent)	Viscous parameter, μ_o (poise)	Yield Stress, τ_o (dyn/cm^2)
1	0.15	0.4871	0.0277
2	0.37	12.4972	0.244298
3	0.4	20.398	0.30596
4	0.45	33.566	0.40873
5	0.54	57.2684	0.593716
6	0.6	73.07	0.71704
7	0.65	86.238	0.81981
8	0.86	141.5436	1.251444
9	1	178.414	1.5392
10	1.3	257.422	2.15582